A SURVEY OF SMALL SQUARES

MAXEY BROOKE
Sweeny, Texas

In a desperate attempt to make ever-larger word squares, logologists have succeeded only in producing monsters such as the ten-by-ten tautonymic word squares featured in the August and November 1973 issues of Word Ways. Is it not time to recall the intrinsic beauty and extreme versatility of smaller word squares -- say, those of size six-by-six or less? This article surveys a wide variety of such squares, taken from past issues of Word Ways, from Dmitri Borgmann's Language on Vacation (Scribner's, 1965), and from Martin Gardner's Mathematical Games column in Scientific American.

A word square is an n-by-n grid of letters spelling n words both horizontally and vertically; a double word square is one in which the horizontal words differ from the vertical ones. Examples of both are given at the right. Double word squares are significantly more difficult to construct. No one has yet succeeded in building a six-by-six double word square using only words from Webster's Pocket Dictionary, despite a cash offer of $50 by William Sunners in the November 1972 Word Ways.

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T R E N D     S C A L E
R O V E R     P A G A N
E V E R Y     I R A T E
N E R V E     T O T E M
D R Y E R     E M E R Y
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If the square forms the same set of words when read from either direction, it is a reversible word square. The grand-daddy of all word squares is the reversible one at the right, found scratched on a fragment of wall plaster from a Roman villa at Cirencester, England. J. Newton Friend, in his book Numbers: Fun and Facts (Scribner's, 1954), translates the Latin as "The mechanic Arepo guides the wheels at work". A special kind of reversible word square is the one whose words are all palindromes: the palindromic word square.

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S A T O R
A R E P O
T E N E T
O P E R A
R O T A S
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Remove the outside letters from a word square. If the remaining letters still form a word square, it is known as a bordered word square.

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A P A R T
P E W E E
A W A R E
R E R A N
T E E N S
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E W E
W A R   A
E R A
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In a progressive word square, the first letter of a word is deleted

and another letter is added at the end to produce the next word in the square. The five-by-five example at the right has been taken from Language on Vation. The cyclic square is a special case of a progressive word square -- in it, each word is formed by placing the first letter of the preceding word at the end. A three-by-three example consisting of very common words is given at the right.

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C A R O M
A R O M A
R O M A N
O M A N I
M A N I C

A T E
T E A
E A T
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In the August 1972 Word Ways, Mary Youngquist presented a four-by-four double word square using only three different letters. On the other hand, George H. Ropes of Scarsdale, N.Y. successfully constructed a four-by-four double word square using sixteen different letters. None of his words were plurals or proper names, and all can be found in Webster's Second. This tour de force was originally printed in the July/August issue of the MIT Technology Review, and was cited in the November 1974 Word Ways.

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E R R E     C Y S T
S E E R     O P A H
S E R E     R I M U
E S E R     F L E D
```

In sequential word squares, a given word is successively placed in each position of a square. For example, the word square sometimes FALLS down ...

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F A L L S   A F A C E   B E F I T   C R O F T   S T A F F
A R E A L   F A L L S   E V A D E   R A D A R   T E R R A
L E A V E   A L L O T   F A L L S   O D D L Y   A R E A L
L A V E D   C L O S E   I D L E S   F A L L S   F R A I L
S L E D S   E S T E R   T E S S A   T R Y S T   F A L L S
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but it always RISES again.

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M A N O R   S C A R P   V E R S T   O R A G E   R I S E S
A L I B I   C U R I O   E L I T E   R I S E S   I D O L A
N I M E S   A R I S E   R I S E S   A S S E T   S O R E L
O B E S E   R I S E S   S T E P S   G E E S E   E L E M I
R I S E S   P O E S Y   T E S S A   E S T E R   S A L I X
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On page 111 of Language on Vacation, Dmitri Borgmann exhibited several three-by-three double word squares in which the nine letters, read off in order, form a word of their own.

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L A C     P A L     P A R
E R A     A V E     A P A
T E D     R E D     T E D
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In the same book, he defined the sentential word square -- one in which the component words, read off in order, make some sort of sense: LEAVE ELLEN ALONE, VENOM ENEMY. J. A. Lindon presented more than 100 five-word sentences of this type in the November 1969 Word Ways. If the word square is reversible, the resulting sentence will be palindromic -- but now it is best to relax the require-

ment that the words of the sentence match the words in the word square: REVEL EVER, EVE! O EVE, REVEL EVER!

In The World's Best Puzzles (1925), reprinted by Scribner's in 1968 under the title 300 Best Word Puzzles, Henry E. Dudeney challenged his readers to construct a three-by-three double word square containing sixteen different words: the six standard words, their six reversals, the two diagonal words, and their two reversals. The best he was able to achieve was a square containing thirteen words, given at the left below. Martin Gardner in the June 1964 Scientific American encouraged his readers to do better, and in October he cited five squares (given below) in which all sixteen words were placed. These squares were contributed by Dmitri Borgmann, Leslie Card, Mrs. D. Harold Johnson, Peter Kugel and Wyile Wilson, respectively.

G E T	A T E	E R A	A T S	S E R	E E L
A I A	R A E	L E E	R I A	T A O	T A O
S U P	T O R	S A N	T A D	A R D	A R T

In the February 1964 issue of Scientific American, Solomon Golomb proposed the name Latin word square for an n-by-n word square using n different letters n times each, arranged in such a way that each letter appears exactly once in each row and column. The cyclic word square is a special case of a Latin word square; however, the Latin word square at the right is not cyclic. This remarkable square, presented in C. C. Bombaugh's Oddities and Curiosities of Words and Literature (Dover reprint, 1961), is Latin both in arrangement and words: TIME (fear thou), ITEM (likewise), METI (to be measured) and EMIT (he buys). (If METI were an English word, this would be a bilingual word square as well.)

T I M E
I T E M
M E T I
E M I T

In the February 1972 issue of Word Ways, Walter Penney constructed the unusual letter square given at the right. It is not a word square in the classical sense because the down entries are not words. However, it is a Latin square in a numerical rather than a literal sense. Taking each of the five row words separately, assign each letter its alphabetic "rank" within the word, ranking from one to five according to alphabetic order. This results in the numerical array given above, which can be recognized as a Latin square in numerical form. It is a remarkable fact that if the letters in the above square are ordered column by column instead of row by row, exactly the same numerical array appears. Since this is "going the whole hog" Penney's letter square has been dubbed a pig-Latin square. In the August 1972 Word Ways, Mary Youngquist presented the six-by-six pig-Latin square given at the top of the next page.

A C T O R	1 2 5 3 4
D E C R Y	2 3 1 4 5
M A R S H	3 1 4 5 2
S T E A M	4 5 2 1 3
W R O N G	5 4 3 2 1

It is possible to construct smaller pig-Latin squares that are word squares as well. The five-by-five pig-Latin word square given here

A M B U S H	S T A L E	C H A R
P E A N U T	T A L E S	H A V E
T R I O D E	A L G O T	A T E N
S A N E L Y	L E O R A	T E R A
C H Y T R A	E S T A R	
I S R A E L		

was also produced by Mary Youngquist and published in the February 1973 Word Ways. In the same issue, Murray Pearce built the four-by-four pig-Latin double word square given at the right.

A magic square is an array of numbers whose members add to the same sum vertically and horizontally. The numerical Latin square given earlier is a magic square; in other magic squares, all the numbers are different. If the letters in each row of a word square are assigned their alphabetical rank, a square array of numbers results; by proper choice of words, it is possible to make a magic square out of them. The square at the right converts into a magic square whose rows and columns all add to 47. (REGOB is a plausible coinage, but is not found in any dictionary.)

C	A	R	E	T	3	1	18	5	20
A	V	E	R	A	1	22	5	18	1
R	E	G	O	B	18	5	7	15	2
E	R	O	D	E	5	18	15	4	5
T	A	B	E	S	20	1	2	5	19

The six-by-six word square at the right was one of the first English word squares to be published, appearing in a letter to the British periodical Notes and Queries on July 21, 1859 by a reader who signed himself W. W. 104 years later, Martin Gardner asked his readers to discover other ways to square the circle. According to the February 1964 Scientific American, he was inundated by over a thousand squares of 200 different types, the commonest given at the right. The most popular second word was INURES, with IBERIA, ICARUS and ISOHEL following in that order.

C	I	R	C	L	E
I	C	A	R	U	S
R	A	R	E	S	T
C	R	E	A	T	E
L	U	S	T	R	E
E	S	T	E	E	M

C	I	R	C	L	E		S	Q	U	A	R	E
I	N	U	R	E	S		Q	I	N	T	A	R
R	U	D	E	S	T		U	N	L	A	C	E
C	R	E	A	S	E		A	T	A	V	I	C
L	E	S	S	E	R		R	A	C	I	S	T
E	S	T	E	R	S		E	R	E	C	T	S

A few readers attempted to square the square as well, but this turned out to be more difficult. The best word square of this type, given above, can be found on p. 303 of Language on Vacation.

Although they are often hard to diagram, word squares can be generalized to more than two dimensions. Possibly the earliest example of a word cube, given below, was constructed by J. A. Lindon and ap-

R A R E	A V I D	R I S E	E D E N
A V I D	V E T O	I T E M	D O M E
R I S E	I T E M	S E M I	E M I R
E D E N	D O M E	E M I R	N E R O

peared in the October 1961 issue of Recreational Mathematics Magazine. The words in the third dimension are read off from corresponding positions in the four squares; thus, the upper left corners spell RARE. Notice that four words (RARE, VETO, SEMI, NERO) are used three times apiece and six words are used six times apiece. In the February 1964 Scientific American, several readers cubed the cube; the following solution by Quentin Derkletterer is typical.

C U B E	U G L Y	B L U E	E Y E S
U G L Y	G L U E	L U L L	Y E L P
B L U E	L U L L	U L U S	E L S E
E Y E S	Y E L P	E L S E	S P E D

Darryl Francis became the first person to construct a triple word cube, in the August and November 1971 issues of Word Ways.

M A L I	O P A L	N O I L	A D D S
I C E S	R O L E	A L M A	N E A T
T E A M	E L S E	S L A T	T A R A
A R T S	S O O T	T A M E	I D E M

Word squares in higher dimensions have also been constructed. In the October 1961 Recreational Mathematics Magazine, J. A. Lindon gives an example of a four-by-four-by-four-by-four word hypercube, and in the August 1971 Word Ways Darryl Francis has constructed a three-by-three-by-three-by-three quadruple word hypercube (that is, a hypercube in which all 108 of the three-letter words are different). Finally, he has also constructed a two-by-two-by-two-by-two-by-two quintuple hyperhypercube, given in the November 1971 Word Ways.

This survey of word squares has been deliberately limited to those of size six-by-six or smaller. It is necessary to violate this restriction to bring to the attention of Word Ways readers two of the finest larger word squares ever constructed. The first, a seven-by-seven double word square constructed by Palmer Peterson of Lennox, South Dakota, was recently called to my attention by Dmitri Borgmann. The second, an eight-by-eight word square constructed by the late Mrs. A. H. Burkholder, has previously been published in several different books. All of the words in both of these squares can be found in the main alphabetical section of Webster's Second Edition. No larger squares are known which are drawn from a single source.

M	I	S	A	V	E	R	
A	D	E	L	I	N	E	
R	O	M	A	N	C	E	
A	L	E	R	C	E	S	
R	I	N	G	E	N	T	
I	N	C	E	N	S	E	
E	G	E	S	T	E	D	

A	G	A	R	I	C	U	S
G	E	N	E	R	A	N	T
A	N	A	C	O	N	D	A
R	E	C	A	N	T	E	R
I	R	O	N	W	O	R	T
C	A	N	T	O	N	A	L
U	N	D	E	R	A	G	E
S	T	A	R	T	L	E	D

Savor these two squares -- you may never see their like again!